Maximum time: 20 *minutes*

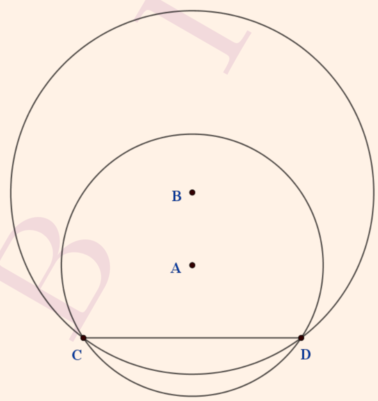
KEM5 - Foundation Maths for Std 9,10 together @ *ABIMS*

Try on your own ! Don't use calculators ! Think and Answer !

Name:______________________________ Standard:____________

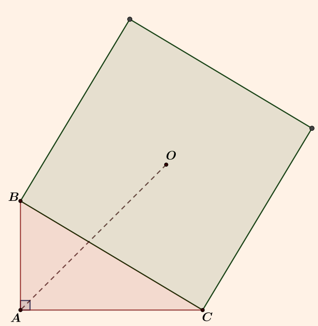
I Answer the following questions accordingly !

1. In the figure, CD is a *common chord of circles* with *centres* A, B as shown.
   If $CD = 24\ cm$, $BC = 20\ cm$ and $AD = 13\ cm$,then $AB =$ ______ cm.



Note: Figure is not drawn to scale!

2. O is the *centre* of a *square* constructed externally on the *hypotenuse* BC of *right triangle* ABC, as shown.



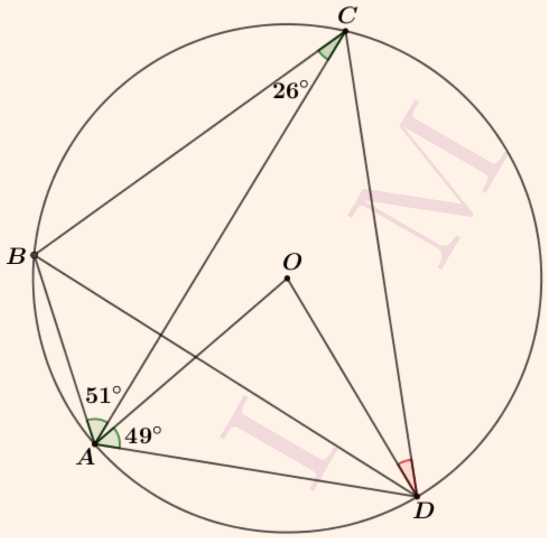
Then, $\angle CAO =$ _____ (*in degrees*).

3. $ABCDEFGH$ is a *cyclic octagon* where $\angle ABC = 151°$; $\angle EFG = 109°$; $\angle GHA = 133°$.

   Then, $\angle CDE =$ ____ (*in degrees*).

4. A, B, C, D are *points* on a *circle* whose *centre* is O.

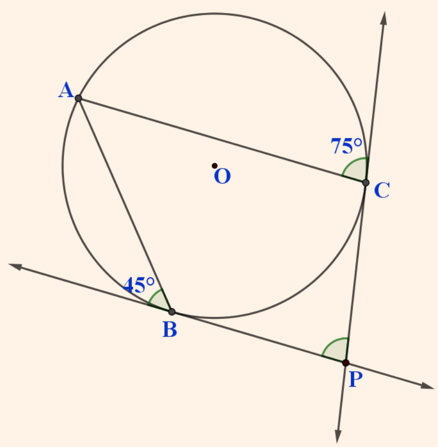
   $\angle OAD = 49°$; $\angle ACB = 26°$; and $\angle CAB = 51°$, as shown.



   The *measure* of $\angle ODC$ = ____ (in *degrees*).

5. From an *exterior point* P, two *tangents* are drawn to a *circle* at B, C as shown.

   O is the *centre* of the *circle*.



   If AB, AC are *chords* of the *circle* making *angles* $45°, 75°$ with the *tangents*,

   then $\angle BPC =$ ____ *(in degrees)*.

Maximum time: 110 *minutes*

Basics from previous standards, Real Numbers, Polynomials
Similar Triangles, Circle Properties, Theorems and Properties

KEM5 - Foundation Maths for Std 9,10 together @ *ABIMS*

Try on your own ! Don't use calculators ! Think and Answer !

Name:______________________________ Standard:__________

I Answer the following questions accordingly !

I.I Section - A : Questions on *Concepts*

1. How many of the following 8 numbers are irrational numbers?

$\sqrt{225}$ $\quad 3\sqrt{18}$ $\quad \sqrt{\dfrac{13}{39}}$ $\quad \sqrt{\dfrac{44}{1331}}$

$-\sqrt{0.9}$ $\quad (3+\sqrt{7}) - \left(\dfrac{22}{7} + \sqrt{7}\right)$ $\quad \sqrt[3]{\dfrac{1536}{375}}$ $\quad 0.2024$

A. 3 B. 4 C. 5 D. 6

2. Which of the following *statements* is definitely TRUE?

A. $\dfrac{a}{b} = \dfrac{a}{c} \Rightarrow b = c$ where a, b, c are *real numbers.*

B. $a^m = a^n \Rightarrow m = n$ where a, m, n are *real numbers.*

C. $a^m = b^m \Rightarrow a = b$ where a, b, m are *real numbers.*

D. none of these.

3. AB is diameter of a circle and C is an *interior point of the circle.*

Which of the following *statements* is definitely TRUE?

A. $\angle ACB$ is *an acute angle.* B. $\angle ACB$ is *an obtuse angle.*

C. $\angle ACB$ is *a right angle.* D. none of these

4. $\triangle ABC \sim \triangle DEF$ such that $\dfrac{Area\ of\ \triangle ABC}{Area\ of\ \triangle DEF} = \dfrac{25}{49}$.

   Then, the ratio of perimeter of $\triangle DEF$ to the perimeter of $\triangle ABC$ will be

   A. $\dfrac{7}{5}$ B. $\dfrac{5}{7}$ C. $\dfrac{10}{3}$ D. none of these

5. Which of the following is a factor $x^3 + 13x^2 + 32x + 20$?

   A. $x^2 + 3x + 2$ B. $x^2 + 12x + 20$

   C. $x^2 + 11x + 10$ D. All of these

6. Which of the following *statements* is definitely *true* ?

   A. The *sum* of *two linear expressions* is *quadratic.*

   B. The *sum* of *two linear expressions* is *linear.*

   C. The *sum* of *two linear expressions* is *quadratic or linear.*

   D. The *sum* of *two linear expressions* is *linear or constant.*

7. If 2 is a zero of the polynomial expression in $x, 2x^2 + 3x - P$, then the value of P is

   A. -4 B. 0 C. 6 D. 14

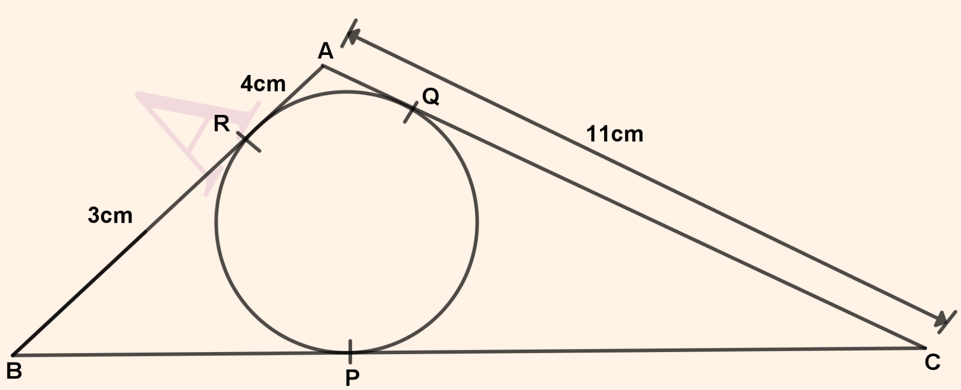
8. Which of the following *equations* is not a *quadratic equation* in x?

   A. $3x^2 + 5x + 7 \ = \ 2x^2 + 5x + 7$ B. $2x^3 + 2x - 7 \ = \ 2\big(x^3 - 5x^2 + 3\big)$.

   C. $x + \dfrac{1}{x} \ = \ \dfrac{41}{20}$ D. $\dfrac{(x-1).(x-4)}{4} + \dfrac{(x-4).(x-5)}{12} + \dfrac{(x-1).(x-5)}{-3} \ = \ 1$

9. In the given figure, a circle is inscribed in $\triangle ABC$.

   P, Q, R are contact points such that $AR = 4\ cm$; $BR = 3\ cm$ and $AC = 11\ cm$, as shown.



   Note: The figure is not drawn to scale.

   The length of BC is

   A. $12\ cm$ B. $10\ cm$ C. $9\ cm$ D. $15\ cm$

10. If $y = \sqrt{\dfrac{5+2\sqrt{6}}{5-2\sqrt{6}}}$, then the value of $y(y-10)$ is

    A. 1 B. -1 C. 0 D. none of these

11. $a^4 + a^2 + 1 \ =$

    A. $\left(a^2+a+1\right).\left(a^2-a+1\right)$ B. $\left(a^2+1\right)^2$

    C. $\left(a+1\right)^2.\left(a^2-a+1\right)$ D. none of these

12. $ABCDEFGH$ is a *regular octagon.* Which of the following is *congruent* to AD?

    A. BG B. CE C. DH D. EF

13. a, b are *real numbers* such that $a + b\sqrt{2} = 3 - 2\sqrt{2}$. Then, $a^2 + b^2 =$

    A. *cannot exactly determine* B. 13 C. 5 D. 25

14. Which of the following is a *trinomial*?

    A. $x^2 + 2x + 3$ B. $4z^3$ C. $y^9 + 1$ D. none of these

15. $81n^2 - 162n + 80 =$

    A. $(27n-10)(3n-8)$ B. $(9n-10)(9n-8)$

    C. $(81n-10)(n-8)$ D. none of these

16. $\triangle ABC$ is right angled triangle with two of its sides as $AB = 3$ *unit* and $BC = 4$ *unit.*

    If the *area* of $\triangle ABC$ is a non-integer measure K *sq. unit.* then the value of $4K^2$ is

    A. 144 B. 63 C. 36 D. none of these

17. Which of the following is *an equation*?

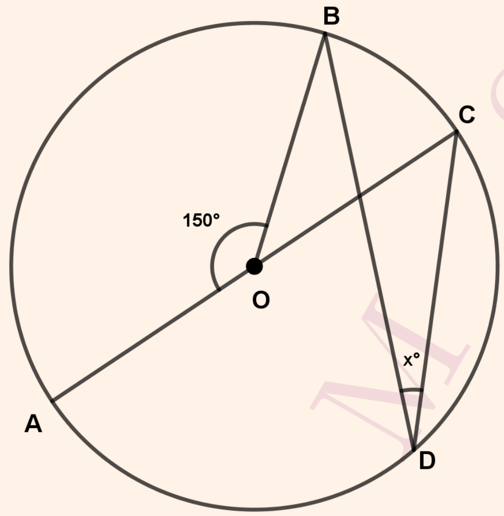
    A. $2+3 \ = \ 6$ B. $-5 < 0$

    C. $2+3 \ \neq \ 1+5$ D. none of these

18. ΔABC and ΔPQR are such that $\dfrac{AB}{PR} \ = \ \dfrac{BC}{QR} \ = \ \dfrac{CA}{PQ}$. Then, $\angle BAC \ =$

    A. $\angle PQR$ B. $\angle PRQ$ C. $\angle QPR$ D. none of these.

19. A, B, C, D are points on the circle whose centre is O; AC being the diameter.



If $\angle AOB = 150°$ and $\angle BDC = x°$, then $x° =$

A. 15° B. 30° C. 75° D. 20°

20. Which of the following is a cubic polynomial,whose three zeros are 2, $\frac{-1}{2}$ and $\frac{1}{3}$?

A. $6x^3-11x^2-3x+2$ B. $6x^3-11x^2+3x+2$ C. $6x^3-11x^2+3x-2$ D. $6x^3+11x^2-3x-2$

I.II Section - B : Questions on *Applications*

21. If BL, CM are medians of $\triangle ABC$ which is right angled at A, then $\frac{BL^2 + CM^2}{BC^2} =$

A. $\frac{4}{5}$ B. $\frac{5}{4}$ C. $\frac{9}{7}$ D. $\frac{7}{9}$

22. If the cubic polynomials $(Kx^3 + 3x^2 - 3)$ and $(2x^3 - 5x + K)$ both leaves the same remainder when divided by $(x - 4)$, then the value of constant K, is

A. 0 B. 1 C. 2 D. -1

23. Let the simplest rational equivalent to $0.\overline{6} + 0.\overline{7} + 0.4\overline{7}$ be $\frac{p}{q}$ where p, q are positive integers.

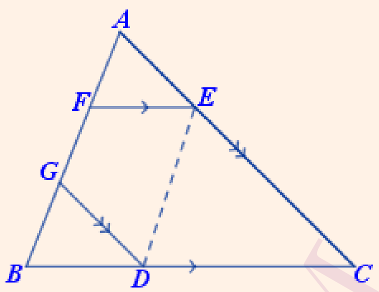
Then the value of $(p - q)$ is

A. 73 B. 77 C. 83 D. 87

SEE NEXT PAGE!

24. In the given figure $\triangle ABC, D, E, F, G$ are points on sides BC, CA, AB, AB respectively.

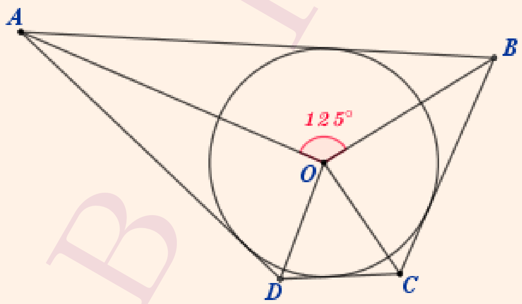
$FE \parallel BC, GD \parallel AC$ as shown



Which of the following is correct?

A. $\dfrac{AF}{FE} = \dfrac{GB}{GD}$ B. $\dfrac{DC}{EC} = \dfrac{AF}{GB}$ C. $\dfrac{AE}{EC} = \dfrac{BD}{DC}$ D. none of these

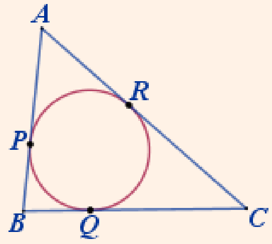
25. O is the centre of a circle that can be inscribed in a quadrilateral $ABCD$, as shown.



If $\angle AOB = 125°$, then $\angle COD =$

A. 55° B. 45° C. 35° D. none of these

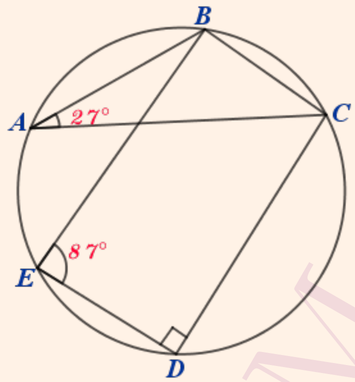
26. The sides AB, BC, CA of $\triangle ABC$ touch a circle at P, Q, R respectively. If $PA = 5\ cm$, $BP = 4\ cm$ and $AC = 13\ cm$, then the length of BC is ______ cm.



Note: Figure is not drawn to scale.

A. 12 B. 10 C. 11 D. none of these

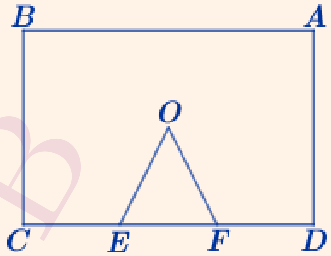
27. A, B, C, D, E are points on a circle such that $\angle BAC = 27°, \angle BED = 87°$ and $\angle CDE = 90°$, as shown.



Then, the measure of $\angle BDE =$

A. $67°$ B. $63°$ C. $72°$ D. $77°$

28. $ABCD$ is a rectangle, whose centre is O. E, F are points on side CD such that $CE = EF = FD$, as shown.



Then, the ratio of $\dfrac{Area\ of\ \triangle OEF}{Area\ of\ rectangle\ ABCD}$ is

A. $\dfrac{1}{9}$ B. $\dfrac{1}{15}$ C. $\dfrac{1}{12}$ D. $\dfrac{1}{8}$

29. If $(x-2)\left(x-\dfrac{1}{2}\right)$ is a factor of Px^2+5x+r, then which of the following is $TRUE$?

A. $P=-r$ B. $P=r$ C. $2P=r$ D. none of these

30. If $y=\dfrac{3a}{5}$, then the value of $\dfrac{\sqrt{a+y}+\sqrt{a-y}}{\sqrt{a+y}-\sqrt{a-y}}$ is

A. 1 B. 2 C. 3 D. none of these

I.III Section - C : Questions on *Applications*

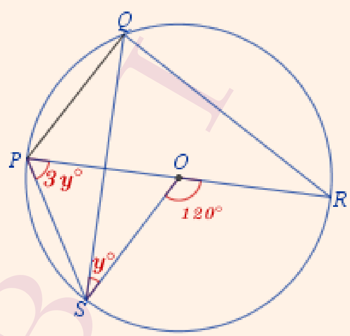
31. $x^2 - 3x + 2$ is a factor of $2x^4 - 5x^3 + Kx^2 - x + 2$ where K is a constant.

    The value of K is ______.

32. If $y = \dfrac{\sqrt{7}}{5}$ and $\dfrac{5}{y} = P\sqrt{7}$, then the length of the recurring part of the rational number P in its decimal representation is ______.

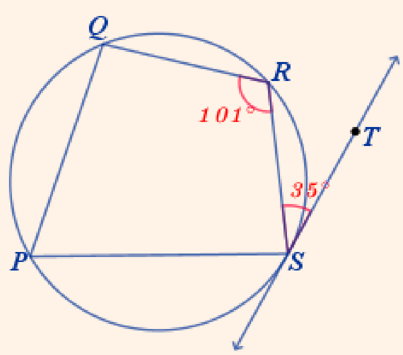
33. $\triangle ABC$ is circumscribed by a circle whose centre is O. PA, PB are tangents from an external point P. If $\angle APB = 80°$ and $\angle AOC = 140°$, then the value of $\angle CAB =$ ______ (*in degrees*).

34. $PQRS$ is a cyclic quadrilateral where PR is the diameter of the circumscribing circle, whose centre is O, as shown.



    If $\angle SOR = 120°, \angle QSO = y°, \angle SPO = 3y°$, then $y =$ ______

35. P, Q, R, S are points on the circumference of a circle. TS is a tangent to the circle at point S, as shown.

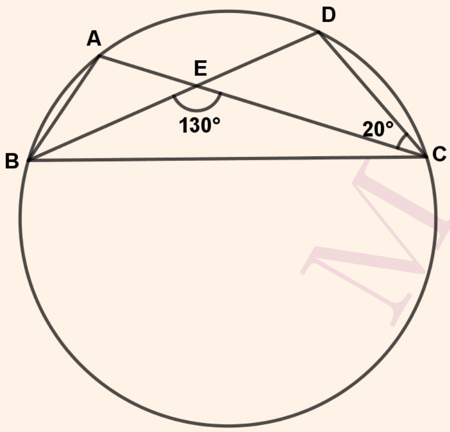


    If $\angle RST = 35°, \angle QRS = 101°$, then $\angle QSR =$ ______ (*in degrees*).

36. *Octal number* $567_8 =$ _______$_5$ in *Quinary System.*

37. In the given figure, A, B, C and D are four points on a circle.

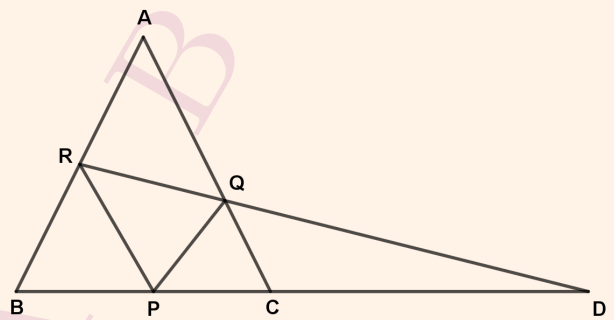
AC and BD intersect at a point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$.



Then, the measure of $\angle BAC$ is ______ (in degrees).

38. In the given $\triangle ABC$, P, Q, R are points on sides BC, CA, AB respectively.

If $PQ \parallel BA$ and $PR \parallel CA$. If $PD = 12$ *cm*, then $BD \times CD =$ ______ cm^2.



Note: D is an exterior point to $\triangle ABC$ where PQ and BC meet, as shown.

39. The *cubic polynomial* $6y^3 + 10y^2 + 5y - 3$ can be *factorised* into *a linear and a quadratic, each with integer coefficients in* y.

Then, the *sum of the coefficients* of the *quadratic expression* is ______.

40. The area of a triangle with side lengths 13 *unit*, 20 *unit*, k *unit* is 126 *sq. unit*, then the value of k is _____.

PRINCIPLE OF ALGEBRAIC FACTORISATION - by Sadagopan Rajesh

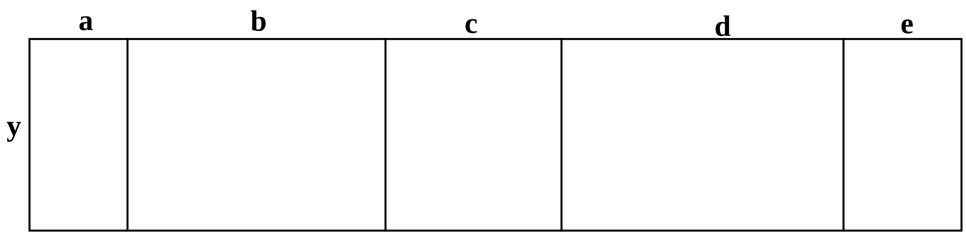
An Algebraic expression which may be in the *sum form* expressed alternately as a *product form of its factors*, is called the *factorised form* of the algebraic expression.

Algebraic expression can be classified into three categories based on its forms.

FORM – I

An Algebraic expression which is in the form of SUM OF PRODUCTS has a common term throughout.

Geometric Version of FORM – I



We observe that the five smaller rectangles having a common width combine to form a larger rectangle.

The sum of the areas of the five rectangles gives the area of the larger rectangle.

INTERNAL ADDITION OF AREAS GIVES EXTERNAL AREA!

INTERNAL SUM OF PRODUCTS GIVES EXTERNAL PRODUCT!

$$ya + yb + yc + yd + ye = y.(a+b+c+d+e) \quad \cdots\cdots \textbf{(1)}$$

SUM FORM (start) $\quad$ PRODUCT FORM (end)

Algebraic Version of FORM – I

$P_1 + P_2 + P_3 + \cdots + P_n$ where C is a common to all the *product terms* $P_1, P_2, P_3, \cdots, P_n$.

$\therefore$ $P_1 + P_2 + P_3 + P_4 + \cdots + P_n$ can be *simplified* and *expressed* as follows:

$$P_1 + P_2 + P_3 + P_4 + \cdots + P_n = C.(R_1 + R_2 + R_3 + R_4 + \cdots + R_n) \quad \cdots\cdots \textbf{(1)}$$

SUM FORM (start) | PRODUCT FORM (end)

Both Left Hand Side form (LHS form) and Right Hand Side form (RHS form) of the *equivalent forms* of the expression (1) involve *addition* (SUM) and *multiplication* (PRODUCT) operations.

Though $P_1, P_2, P_3, \cdots, P_n$ are *product terms* in the LHS of (1), the *initial expression* $P_1 + P_2 + P_3 + P_4 + \cdots + P_n$ is nothing but SUM OF PRODUCTS FORM, where SUM (*addition*) is dominant to PRODUCT (*multiplication*).

Therefore, LHS of (1) is a SUM FORM.

Though $R_1, R_2, R_3, \cdots, R_n$ are added in the RHS of (1), the *final expression* $C.(R_1 + R_2 + R_3 + R_4 + \cdots + R_n)$ is nothing but PRODUCT OF SUM FORM, where PRODUCT (*multiplication*) is dominant to SUM (*addition*).

Therefore, RHS of (1) is a PRODUCT FORM, which is the *final factorized form.*

Few Examples:

$ax + ay + az + ap + aq$ $= a.x + a.y + a.z + a.p + a.q$ $= a.(x + y + z + p + q)$	$10x + 15$ $= 5 \times 2x + 5 \times 3$ $= 5.(2x + 3)$	$x^3 + 4x^2 + 7x$ $= x.x^2 + x.4x + x.7$ $= x.(x^2 + 4x + 7)$

Initial Expression	FORM	Type of FORM	*All common term*	Factoring	FORM of Final Expression
$ax + ay + az + ap + aq$	SUM	FORM – I	a	$ax + ay + az + ap + aq$ $= a.(x + y + z + p + q)$	PRODUCT
$10x + 15$	SUM	FORM – I	5	$10x + 15 = 5.(2x + 3)$	PRODUCT
$x^3 + 4x^2 + 7x$	SUM	FORM – I	x	$x^3 + 4x^2 + 7x$ $= x.(x^2 + 4x + 7)$	PRODUCT

Steps to factorise FORM – I expressions.

Step 1: Check whether the algebraic expression is in SUM of PRODUCT FORM.

Algebraic Expression	SUM of PRODUCT FORM
$12a^4b^2c^4 - 20a^5c^3 + 24a^6bc$	✓

Step 2: How the expression is made up?

The expression is made of *product of number, unknowns a,b and c.*

Step 3: What are the numbers in the product terms? What is common to all?

The numbers in the product terms are $12, -20$ and 24.

4 is common to all the numbers in the product terms.

Algebraic Expression	NUMBER IN THE			COMMON NUMBER IN THE EXPRESSION
	First Product Term	Second Product Term	Third Product Term	
$12a^4b^2c^4 - 20a^5c^3 + 24a^6bc$	12	-20	24	4

Step 4: What are the unknowns in the product terms having a? What is common to all?

The unknowns involving a in the product terms are a^4, a^5 and a^6.

a^4 is common to all unknowns involving a in the product terms.

Algebraic Expression	TERMS INVOLVING a IN THE			COMMON TERM INVOLVING a IN THE EXPRESSION
	First Product Term	Second Product Term	Third Product Term	
$12a^4b^2c^4 - 20a^5c^3 + 24a^6bc$	a^4	a^5	a^6	a^4

Step 5: What are the unknowns in the product terms having b? What is common to all?

The unknowns involving b are present in the first and third product terms but absent in the second product term.

Therefore, there is no common to all unknowns involving b in the product terms.

Algebraic Expression	TERMS INVOLVING b IN THE			COMMON TERM INVOLVING b IN THE EXPRESSION
	First Product Term	Second Product Term	Third Product Term	
$12a^4b^2c^4 - 20a^5c^3 + 24a^6bc$	b^2	$--$	b	*NO TERM*

Step 6: What are the unknowns in the product terms having c? What is common to all?

The unknowns involving c in the product terms are c^4, c^3 and c.

c is common to all unknowns involving c in the product terms.

Algebraic Expression	TERMS INVOLVING c IN THE			COMMON TERM INVOLVING c IN THE EXPRESSION
	First Product Term	Second Product Term	Third Product Term	
$12a^4b^2c^4 - 20a^5c^3 + 24a^6bc$	c^4	c^3	c	c

Step 7: Is the algebraic expression is in FORM – I ?

YES !

Step 8: Factorise the expression as follows:

$$\begin{aligned}&12a^4b^2c^4 - 20a^5c^3 + 24a^6bc\\&= 4a^4c.3b^2c^3 - 4a^4c.5ac^2 + 4a^4c.6a^2b\\&= 4a^4c.\left(3b^2c^3 - 5ac^2 + 6a^2b\right)\end{aligned}$$

One more example:

$$\begin{aligned}&x^4y^3z - x^2y^5z^3 + x^7y^4z^2\\&= x^2y^3z.x^2 - x^2y^3z.y^2z^2 + x^2y^3z.x^5yz\\&= x^2y^3z.\left(x^2 - y^2z^2 + x^5yz\right)\end{aligned}$$

Factorise each of the following FORM – I expressions:

1) $a^2 + ab$ 2) $a^3 + a^2b + ab^2$ 3) $x^2 + 3x$ 4) $7x - x^2$

5) $2abx - 2ab^2 - 2a^2b$ 6) $y^2 - 6y$ 7) $x^2y - y^3 - z^2y$ 8) $2ax^2y - 4ax^2z$

9) $4x^2y + 6x^3y^2z$ 10) $ayx + yx^3 - 2y^2x^2$ 11) $16a - 2a^2$ 12) $120m^2 + 180$

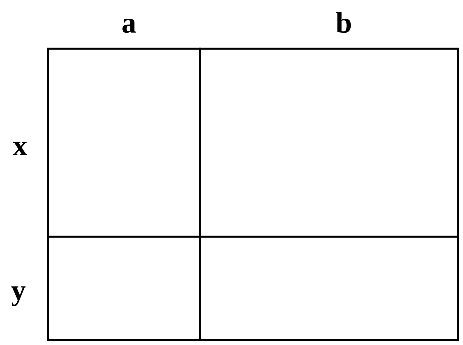
13) $cb^2a - c^2ba^3 + 2cba^2$ 14) $8a^2b^3c + 20ab^2c^5 + 24a^5bc^3$

15) $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4$

FORM – II

An Algebraic expression which is in the form of SUM OF PRODUCTS has no common term throughout. But it can be expressed as SUM OF FORM – I GROUPS such that each group can be factored. The commons taken out from each group may be different but the remains become common.

Geometric Version of FORM – II



We observe four smaller rectangles with no common width, form a larger rectangle.

The sum of the areas of the four rectangles gives the area of the larger rectangle.

INTERNAL ADDITION OF AREAS GIVES EXTERNAL AREA!

INTERNAL SUM OF GROUPED PRODUCTS GIVES EXTERNAL PRODUCT!

$$xa + xb + ya + yb = (x+y).(a+b) \quad \cdots\cdots \ \textbf{(2)}$$

SUM FORM (start) PRODUCT FORM (end)

Algebraic Version of FORM – II

$P_1 + P_2 + P_3 + \cdots + P_n$ where there is <u>no common to all</u> *product terms* $P_1, P_2, P_3, \cdots, P_n$.

$\therefore$ $P_1 + P_2 + P_3 + P_4 + \cdots + P_n$ can be *simplified* and *expressed* as follows:

$$\begin{aligned} & P_1 + P_2 + P_3 + P_4 + \cdots + P_n \\ &= G_1 + G_2 + G_3 + \cdots + G_m \\ &= C_1.R + C_2.R + C_3.R + \cdots + C_m.R \\ &= (C_1 + C_2 + C_3 + \cdots + C_m).R \end{aligned}$$

: Here the REMAINS OF THE GROUPS becomes COMMON THROUGH OUT!

Here are few illustrations of factorization of FORM – II expressions:

$$
\begin{aligned}
& ap+aq+bp+bq \\
&=(ap+aq)+(bp+bq) \\
&=a.(p+q)+b.(p+q) \\
&=(p+q).(a+b)
\end{aligned}
$$

$$
\begin{aligned}
& x^2+7x+10 \\
&=\left(x^2+5x\right)+(2x+10) \\
&=x.(x+5)+2.(x+5) \\
&=x.\mathrm{C}+2.\mathrm{C} \qquad \textit{where C is the common term}; \ \mathrm{C}=x+5 \\
&=\mathrm{C}.(x+2) \\
&=(x+5).(x+2)
\end{aligned}
$$

Steps to factorise FORM – II expressions.

Step 1: Check whether the algebraic expression is in SUM of PRODUCT FORM.

Algebraic Expression	SUM of PRODUCT FORM
$ax+ay+bx+by$	✓

Step 2: How the expression is made up?

The expression is made of *unknowns a, b, x and y.*

Step 3: Is there a common term throughout? Or Is the expression in FORM – I type?

NO!

Step 4: Can the algebraic expression be rewritten as SUM of FORM – I GROUPS?

YES!

Step 5: Rewrite the algebraic expression as SUM of FORM – I GROUPS.

Algebraic Expression	SUM of FORM – I GROUPS
$ax+ay+bx+by$	$(ax+ay)+(bx+by)$

Step 6: What is common in the *first* FORM – I GROUP?

What is remains in the *first* FORM – I GROUP?

Algebraic Expression	*First* FORM – I GROUP	FORM – I GROUP	
		Common term	Remains
$(ax+ay)+(bx+by)$	$(ax+ay)$	a	$(x+y)$

Step 7: What is common in the *second* FORM – I GROUP?

What is remains in the *second* FORM – I GROUP?

Algebraic Expression	*Second* FORM – I GROUP	FORM – I GROUP	
		Common term	Remains
$(ax+ay)+(bx+by)$	$(bx+by)$	b	$(x+y)$

Step 8: Is the ***remains*** of the FORM – I GROUPS of the expression same?

YES! Therefore, the algebraic expression is in FORM – II.

Step 9: Factorise the expression as follows:

$$
\begin{aligned}
& ax+ay+bx+by \\
& =(ax+ay)+(bx+by) \\
& =a.(x+y)+b.(x+y) \\
& =(x+y).(a+b)
\end{aligned}
$$

One more example:

$$
\begin{aligned}
& x^3-2x^2+1 \\
& =x^3-x^2-x^2+x-x+1 \\
& =\left(x^3-x^2\right)-\left(x^2-x\right)-(x-1) \\
& =x^2.(x-1)-x.(x-1)-1.(x-1) \\
& =(x-1).\left(x^2-x-1\right)
\end{aligned}
$$

Factorise each of the following FORM – II expressions:

1) $ax+ay+az+bx+by+bz$
2) $ax+cy+bz+ay+bx+cz+az+by+cx$
3) $6ax-3bx+2ay-by$
4) $a^2+2ab+b^2+3a+3b$
5) $ah-ak+bh-bk$
6) $5ax-15ay-2bx+6by$
7) $x^2+3x+bx+3b$
8) $ms+2mt^2-ns-2nt^2$
9) $x^2y+7x+xy^2+7y$
10) $xy+x+y+1$
11) $xy-12+4x-3y$
12) $xyz+x+y+z+xy+yz+zx+1$

Factorise each of the following <u>FORM – II</u> quadratic expressions:

1) x^2+5x+6 2) x^2+5x-6 3) x^2-5x+6 4) x^2-5x-6

5) x^2+6x+8 6) x^2-6x+8 7) x^2+6x-7 8) x^2-6x-7

9) $x^2+7x+12$ 10) $x^2+10x+24$ 11) $x^2-10x+24$ 12) $x^2-10x-24$

13) $x^2+10x-24$ 14) $x^2+20x+19$ 15) $x^2+45x+126$ 16) $x^2-17x-480$

17) $x^2+47x+496$ 18) $x^2-25x+156$ 19) $x^2+45x+486$ 20) $x^2-40x+204$

Factorise each of the following <u>FORM – II</u> quadratic expressions:

1) $4x^2-15x+9$ 2) $4x^2+37x+9$ 3) $2x^2+11x+9$ 4) $6x^2-5x-6$

5) $3x^2+13x+4$ 6) $4x^2+13x+3$ 7) $12x^2+13x+1$ 8) $11x^2-9x-2$

9) $15x^2-19x+6$ 10) $4x^2-25x+25$ 11) $4x^2-15x-25$ 12) $6x^2-67x-35$

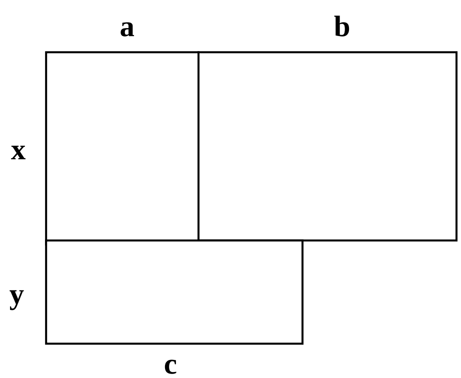
13) $24x^2+481x+20$ 14) $15x^2+34x+15$ 15) $126x^2+45x+1$ 16) $9x^2-34x+25$

17) $9x^2+82x+9$ 18) $5x^2-28x+23$ 19) $16x^2+14x-15$ 20) $-16x^2-14x+15$

FORM – III

An Algebraic expression which is neither in the FORM – I nor FORM – II or otherwise, the one that cannot be factorized is said to be in FORM – III.

Geometric Version of FORM – III



Note: Factorisation is nothing but the product of FORM – III expressions.

ALL THE BEST

Factorisation of FORM-I expressions

1) $a^2+ab = a.a+a.b = a.(a+b)$

2) $a^3+a^2b+ab^2 = a.a^2+a.ab+a.b^2$

$= a.(a^2+ab+b^2)$

3) $x^2+3x = x.x+x.3 = x.(x+3)$

4) $7x-x^2 = x.7-x.x = x.(7-x)$

5) $2abx-2ab^2-2a^2b = 2ab.x-2ab.b-2ab.a$

$= 2ab.(x-b-a)$

6) $y^2-6y = y.y-y.6 = y.(y-6)$

7) $x^2y-y^3-z^2y = y.x^2-y.y^2-y.z^2$

$= y.(x^2-y^2-z^2)$

8) $2ax^2y-4ax^2z = 2ax^2.y-2ax^2.2z$

$= 2ax^2.(y-2z)$

9) $4x^2y+6x^3y^2z = 2x^2y.2+2x^2y.3xyz$

$= 2x^2y.(2+3xyz)$

10) $ayx+yx^3-2y^2x^2 = yx.a+yx.x^2+yx.(-2yx)$

$= yx.(a+x^2-2yx)$

11) $16a - 2a^2 = 2a \cdot 8 - 2a \cdot a = 2a \cdot (8-a)$

12) $120m^2 + 180 = 60 \cdot 2m^2 + 60 \cdot 3 = 60 \cdot (2m^2 + 3)$

13) $cb^2a - c^2ba^3 + 2cba^2$

$= cba \cdot b - cba \cdot ca^2 + cba \cdot 2a$

$= cba \cdot (b - ca^2 + 2a)$

14) $8a^2b^3c + 20ab^2c^5 + 24a^5bc^3$

$= 4abc \cdot 2ab^2 + 4abc \cdot 5bc^4 + 4abc \cdot 6a^4c^2$

$= 4abc \cdot (2ab^2 + 5bc^4 + 6a^4c^2)$

15) $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4$

$= a \cdot a^4 + a \cdot 5a^3b + a \cdot 10a^2b^2 + a \cdot 10ab^3 + a \cdot 5b^4$

$= a \cdot (a^4 + 5a^3b + 10a^2b^2 + 10ab^3 + 5b^4)$

Factorisation of Form-II expressions

1) $ax + ay + az + bx + by + bz$

$= (ax + ay + az) + (bx + by + bz)$

$= a \cdot (x+y+z) + b \cdot (x+y+z)$

$= (x+y+z) \cdot (a+b)$

Alternate method

$ax + ay + az + bx + by + bz$

$= ax + ay + az + bx + by + bz$

$= (ax + bx) + (ay + by) + (az + bz)$

$= x \cdot (a+b) + y \cdot (a+b) + z \cdot (a+b)$

$= (a+b) \cdot (x+y+z)$

Note: ✓ _ · are used to indicate proper grouping.

2) $ax + cy + bz + ay + bx + cz + az + by + cx$

$= ax + cy + bz + ay + bx + cz + az + by + cx$

$= (ax + bx + cx) + (cy + ay + by) + (bz + cz + az)$

$= x \cdot (a+b+c) + y \cdot (c+a+b) + z \cdot (b+c+a)$

$= (a+b+c) \cdot (x+y+z)$

Note: ✓ _ · are used to indicate proper grouping.

$\boxed{\text{Alternate method}}$

$ax + cy + bz + ay + bx + cz + az + by + cx$

$= ax + cy + bz + ay + bx + cz + az + by + cx$

$= (ax + ay + az) + (cy + cz + cx) + (bz + bx + by)$

$= a \cdot (x+y+z) + c \cdot (y+z+x) + b \cdot (z+x+y)$

$= (x+y+z) \cdot (a+c+b)$

Note: ✓ _ · are used to indicate proper grouping.

3) $6ax - 3bx + 2ay - by$

$= (6ax - 3bx) + (2ay - by)$

$= [3x \cdot 2a - 3x \cdot b] + [y \cdot 2a - y \cdot b]$

$= 3x \cdot (2a - b) + y \cdot (2a - b)$

$= (2a - b) \cdot (3x + y)$

$\boxed{\text{Alternate method}}$

$6ax - 3bx + 2ay - by$

$= 6ax - \underline{3bx} + 2ay - \underline{by}$

$= (6ax + 2ay) - (3bx + by)$

$= [2a \cdot 3x + 2a \cdot y] - [b \cdot 3x + b \cdot y]$

$= 2a \cdot (3x + y) - b \cdot (3x + y)$

$= (3x + y) \cdot (2a - b)$

4) $a^2 + 2ab + b^2 + 3a + 3b$

$= (a^2 + 2ab + b^2) + (3a + 3b)$

$= (a+b)^2 + 3(a+b)$

$= (a+b) \cdot [a + b + 3]$

$\boxed{\text{Alternate method}}$

$a^2 + 2ab + b^2 + 3a + 3b$

$= a^2 + ab + ab + b^2 + 3a + 3b$

$= (a^2 + ab) + (ab + b^2) + (3a + 3b)$

$= a.(a+b) + b.(a+b) + 3.(a+b)$

$= (a+b).(a+b+3)$

$\boxed{\text{Alternate method}}$

$a^2 + 2ab + b^2 + 3a + 3b$

$= a^2 + ab + ab + b^2 + 3a + 3b$

$= \overset{\checkmark}{a^2} + \underline{ab} + \overset{\checkmark}{ab} + \underline{b^2} + 3\overset{\checkmark}{a} + \underline{3b}$

$= (a^2 + ab + 3a) + (ab + b^2 + 3b)$

$= [a.a + a.b + a.3] + [b.a + b.b + b.3]$

$= a.(a+b+3) + b.(a+b+3)$

$= (a+b+3).(a+b)$

Note: ✓ — are used to indicate proper grouping.

5) $ah - ak + bh - bk$

$= (ah - ak) + (bh - bk)$

$= a.(h-k) + b.(h-k)$

$= (h-k).(a+b)$

Alternate method

$$ah - ak + bh - bk$$

$$= ah - \underline{ak} + bh - \underline{bk}$$

$$= (ah + bh) - (ak + bk)$$

$$= h.(a+b) - k.(a+b)$$

$$= (a+b).(h-k)$$

Note: ✓ _ are used to indicate proper grouping.

6) $5ax - 15ay - 2bx + 6by$

$$= (5ax - 15ay) - (2bx - 6by)$$

$$= [5a.x - 5a.3y] - [2b.x - 2b.3y]$$

$$= 5a.(x-3y) - 2b.(x-3y)$$

$$= (x-3y).(5a-2b)$$

Alternate method

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$$5ax - 15ay - 2bx + 6by$$

$$= 5ax - 15ay - 2bx + 6by$$

$$= (5ax - 2bx) - (15ay - 6by)$$

$$= [x.5a - x.2b] - [3y.5a - 3y.2b]$$

$$= x.(5a-2b) - 3y.(5a-2b) = (5a-2b)(x-3y)$$

Note: ✓ · are used to indicate proper grouping.

7) $x^2 + 3x + bx + 3b$

$= (x^2 + 3x) + (bx + 3b)$

$= [x \cdot x + x \cdot 3] + [b \cdot x + b \cdot 3]$

$= x \cdot (x + 3) + b \cdot (x + 3)$

$= (x + 3) \cdot (x + b)$

$\boxed{\text{Alternate method}}$

$x^2 + 3x + bx + 3b$

$= \overset{\checkmark}{x^2} + \underline{3x} + \overset{\checkmark}{bx} + \underline{3b}$

$= (x^2 + bx) + (3x + 3b)$

$= x \cdot (x + b) + 3 \cdot (x + b)$

$= (x + b) \cdot (x + 3)$

8) $ms + amt^2 - ns - ant^2$

$= (ms + amt^2) - (ns + ant^2)$

$= [m \cdot s + m \cdot at^2] - [n \cdot s + n \cdot at^2]$

$= m \cdot (s + at^2) - n \cdot (s + at^2)$

$= (s + at^2) \cdot (m - n)$

Alternate method

$$ms + 2mt^2 - ns - 2nt^2$$

$$= \overset{\checkmark}{m s} + 2\dot{m}t^2 - n\overset{\checkmark}{s} - 2\dot{n}t^2$$

$$= (ms - ns) + (2mt^2 - 2nt^2)$$

$$= [s.m - s.n] + [2t^2.m - 2t^2.n]$$

$$= s.(m-n) + 2t^2.(m-n)$$

$$= (m-n).(s+2t^2).$$

Note: ✓ . are used to indicate proper grouping.

9) $x^2y + 7x + xy^2 + 7y$

$$= (x^2y + 7x) + (xy^2 + 7y)$$

$$= [x.xy + x.7] + [y.xy + y.7]$$

$$= x.(xy+7) + y.(xy+7)$$

$$= (xy+7).(x+y)$$

Alternate method

$$x^2y + 7x + xy^2 + 7y$$

$$= x^2\overset{\checkmark}{y} + \underline{7x} + x\overset{\checkmark}{y}^2 + \underline{7y}$$

$$= (x^2y + xy^2) + (7x + 7y)$$

$$= [xy.x + xy.y] + [7.x + 7.y]$$

$$= xy.(x+y) + 7.(x+y)$$

$$= (x+y).(xy+7)$$

10) $xy + x + y + 1$

$$= (xy + x) + (y + 1)$$

$$= [x.y + x.1] + [y + 1]$$

$$= x.(y+1) + 1.(y+1)$$

$$= (y+1).(x+1)$$

$\boxed{\text{Alternate method}}$

$$xy + x + y + 1$$

$$= \overset{\checkmark}{x}y + \dot{x} + \overset{\checkmark}{y} + \dot{1}$$

$$= (xy + y) + (x + 1)$$

$$= [x.y + 1.y] + [x + 1]$$

$$= y.(x+1) + 1.(x+1)$$

$$= (x+1).(y+1)$$

Note: $\checkmark$. are used to indicate proper grouping.

11) $xy - 12 + 4x - 3y$

$= xy - 12 + 4x - 3y$

$= (xy + 4x) - (3y + 12)$

$= x \cdot (y + 4) - 3 \cdot (y + 4)$

$= (y + 4) \cdot (x - 3)$

$\boxed{\text{Alternate method}}$

$xy - 12 + 4x - 3y$

$= xy - 12 + 4x - 3y$

$= (xy - 3y) + (4x - 12)$

$= y \cdot (x - 3) + 4 \cdot (x - 3)$

$= (x - 3) \cdot (y + 4)$

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Note: ✓ • are used to indicate proper grouping.

12) $xyz + x + y + z + xy + yz + zx + 1$

$= xyz + x + y + \underline{z} + xy + yz + zx + \underline{1}$

$= (xyz + xy) + (x + zx) + (y + yz) + (z + 1)$

$= (xyz + xy) + (zx + x) + (zy + y) + (z + 1)$

$= xy \cdot (z + 1) + x \cdot (z + 1) + y \cdot (z + 1) + 1 \cdot (z + 1)$

$$= (z+1).\left[xy + x + y + 1\right]$$

$$= (z+1).\left[(xy + x) + (y+1)\right]$$

$$= (z+1).\left[x.(y+1) + 1.(y+1)\right]$$

$$= (z+1).(y+1).(x+1)$$

$\boxed{\text{Alternate method}}$

$$xyz + x + y + z + xy + yz + zx + 1$$

$$= xyz + x + y + \underline{z} + xy + yz + \underline{zx} + 1$$

$$= (xyz + yz) + (x+1) + (y + xy) + (z + zx)$$

$$= (xyz + yz) + (x+1) + (xy + y) + (zx + z)$$

$$= yz.(x+1) + 1.(x+1) + y.(x+1) + z.(x+1)$$

$$= (x+1).\left[yz + 1 + y + z\right]$$

$$= (x+1).\left[yz + 1 + y + z\right]$$

$$= (x+1).\left[(yz + y) + (1 + z)\right]$$

$$= (x+1).\left[(yz+y) + (z+1)\right]$$

$$= (x+1).\left[y.(z+1) + 1.(z+1)\right]$$

$$= (x+1).(z+1).(y+1)$$

$$= (z+1).\left[xy + x + y + 1\right]$$

$$= (z+1).\left[(xy+x) + (y+1)\right]$$

$$= (z+1).\left[x.(y+1) + 1.(y+1)\right]$$

$$= (z+1).(y+1).(x+1)$$

$\boxed{\text{Alternate method}}$

$$xyz + x + y + z + xy + yz + zx + 1$$

$$= xyz + x + y + \underline{z} + xy + yz + \underline{zx} + 1$$

$$= (xyz + yz) + (x+1) + (y + xy) + (z + zx)$$

$$= (xyz + yz) + (x+1) + (xy + y) + (zx + z)$$

$$= yz.(x+1) + 1.(x+1) + y.(x+1) + z.(x+1)$$

$$= (x+1).\left[yz + 1 + y + z\right]$$

$$= (x+1).\left[yz + 1 + y + z\right]$$

$$= (x+1).\left[(yz+z) + (1+y)\right]$$

$$= (x+1).\left[(yz+z) + (y+1)\right]$$

$$= (x+1).\left[z(y+1) + 1(y+1)\right]$$

$$= (x+1).(y+1).(z+1)$$

Note: ✓ • _ ✓✓ are used to indicate proper grouping.